

Technical Report ARAEW-TR-06028

Location of Impulsive Acoustic Sources in Urban Environments Using Finite-Difference, Time-Domain Modeling of Time Reversal with Data from Small Sensor Arrays

Mark E. Todaro

DECEMBER 2006



ARMAMENT RESEARCH, DEVELOPMENT AND ENGINEERING CENTER
Armaments Engineering & Technology Center
Weapon Systems & Technology



Approved for public release; distribution is unlimited.

Report Documentation Page			Form Approved OMB No. 0704-0188		
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE DEC 2006		2. REPORT TYPE FINAL		3. DATES COVERED 00-00-2004 to 00-00-2005	
4. TITLE AND SUBTITLE Location of Impulsive Acoustic Sources in Urban Environments Using Finite-Difference, Time-Domain Modeling of Time Reversal with Data from Small Sensor Arrays			5a. CONTRACT NUMBER		
			5b. GRANT NUMBER		
			5c. PROGRAM ELEMENT NUMBER		
6. AUTHOR(S) Mark Todaro			5d. PROJECT NUMBER		
			5e. TASK NUMBER		
			5f. WORK UNIT NUMBER		
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. ARMY ARDEC, Benet Laboratories, RDAR-WSB, Watervliet, NY, 12198			8. PERFORMING ORGANIZATION REPORT NUMBER ARAEW-TR-06028		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. ARMY ARDEC, Benet Laboratories, RDAR-WSB, Watervliet, NY, 12198			10. SPONSOR/MONITOR'S ACRONYM(S)		
			11. SPONSOR/MONITOR'S REPORT NUMBER(S)		
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT The author studied the use of time reversal processing with small sensor arrays, less than ten meters across, to locate impulsive sound sources in an urban environment. Sound propagation from the source was modeled in two dimensions using a finite-difference, time-domain (FDTD) method. The model produced simulated pressure-versus-time traces at sensor locations for various sensor arrays. These pressure traces were then reversed in time and fed back into the FDTD model to produce a simulated time-reversed wave. In this manner, a number of sensor arrays were modeled in various hypothetical urban environments to identify the limitations of using time reversal to locate an impulsive source with the smallest arrays possible. Generally, arrays of eight or more sensors with a total spatial extent of eight to ten meters were sufficient for locating non-line-of-sight sources, whereas smaller arrays proved inadequate.					
15. SUBJECT TERMS Sound propagation, acoustic modeling, finite-difference, time-domain FDTD modeling, sensor arrays, urban environments					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT Same as Report (SAR)	18. NUMBER OF PAGES 25	19a. NAME OF RESPONSIBLE PERSON
a. REPORT unclassified	b. ABSTRACT unclassified	c. THIS PAGE unclassified			

The views, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

The citation in this report of the names of commercial firms or commercially available products or services does not constitute official endorsement by or approval of the U.S. Government.

Destroy this report when no longer needed by any method that will prevent disclosure of its contents or reconstruction of the document. Do not return to the originator.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave Blank)		2. REPORT DATE December 2006		3. REPORT TYPE AND DATES COVERED Final
4. TITLE AND SUBTITLE Location of Impulsive Acoustic Sources in Urban Environments Using Finite-Difference, Time-Domain Modeling of Time Reversal with Data from Small Sensor Arrays			5. FUNDING NUMBERS	
6. AUTHORS Mark E. Todaro				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army ARDEC, AETC, WS&T Benet Laboratories, AMSRD-AAR-AEW, B. 40 1 Buffington Street Watervliet Arsenal, NY 12189-4050			8. PERFORMING ORGANIZATION REPORT NUMBER ARAEW-TR-06028	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) U.S. Army ARDEC, AETC, WS&T Benet Laboratories, AMSRD-AAR-AEW, B. 40 1 Buffington Street Watervliet Arsenal, NY 12189-4050			10. SPONSORING / MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) <p>The author studied the use of time reversal processing with small sensor arrays, less than ten meters across, to locate impulsive sound sources in an urban environment. Sound propagation from the source was modeled in two dimensions using a finite-difference, time-domain (FDTD) method. The model produced simulated pressure-versus-time traces at sensor locations for various sensor arrays. These pressure traces were then reversed in time and fed back into the FDTD model to produce a simulated time-reversed wave. In this manner, a number of sensor arrays were modeled in various hypothetical urban environments to identify the limitations of using time reversal to locate an impulsive source with the smallest arrays possible. Generally, arrays of eight or more sensors with a total spatial extent of eight to ten meters were sufficient for locating non-line-of-sight sources, whereas smaller arrays proved inadequate.</p>				
14. SUBJECT TERMS Sound propagation, acoustic modeling, finite-difference, time-domain (FDTD) modeling, sensor arrays, urban environments			15. NUMBER OF PAGES 24	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT Unclassified	

Location of Impulsive Acoustic Sources in Urban Environments Using Finite-Difference, Time-Domain Modeling of Time Reversal with Data from Small Sensor Arrays

Mark E. Todaro, US Army Armament, Research, Development, and Engineering Center,
Benét Laboratories, Watervliet NY 12189-4000

ABSTRACT

The author studied the use of time reversal processing with small sensor arrays, less than ten meters across, to locate impulsive sound sources in an urban environment. Sound propagation from the source was modeled in two dimensions using a finite-difference, time-domain (FDTD) method. The model produced simulated pressure-versus-time traces at sensor locations for various sensor arrays. These pressure traces were then reversed in time and fed back into the FDTD model to produce a simulated time-reversed wave. In this manner, a number of sensor arrays were modeled in various hypothetical urban environments to identify the limitations of using time reversal to locate an impulsive source with the smallest arrays possible. Generally, arrays of eight or more sensors with a total spatial extent of eight to ten meters were sufficient for locating non-line-of-sight sources, whereas smaller arrays proved inadequate.

TABLE OF CONTENTS

INTRODUCTION AND APPROACH-----	1
Time Reversal -----	1
Finite-Difference, Time-Domain Modeling -----	3
RESULTS -----	7
CONCLUSION -----	17
REFERENCES -----	19

FIGURES

Figure 1. Idealized time reversal-----	2
Figure 2. Two-dimensional staggered Yee mesh -----	4
Figure 3. Waveform used for impulsive sound source. -----	6
Figure 4. Calculated acoustic pulse propagating from a point source in a 92 meter by 68 meter hypothetical urban area. -----	7
Figure 5. Time-reversed sound from seven-sensor hexagonal array. -----	8
Figure 6. Time-reversed sound from an array of two small subarrays ten meters apart. -----	9
Figure 7. Time-reversed sound from a five-sensor portion of the seven-sensor hexagonal array backed against a wall.-----	10
Figure 8. Calculated acoustic pulse propagating from a point source at a different location from previous examples. -----	11
Figure 9. Time-reversed sound from the seven-sensor hexagonal array. -----	11
Figure 10. Time reversed sound from the array of two small subarrays ten meters apart. -----	12
Figure 11. Calculated acoustic pulse propagating from a point source at a third location. -----	12
Figure 12. Time-reversed sound from the seven-sensor hexagonal array. -----	12
Figure 13. Time-reversed sound from the array of two small subarrays ten meters apart.-----	13
Figure 14. Calculated acoustic pulse propagating from a line-of-sight point source. -----	13
Figure 15. Time-reversed sound from the seven-sensor hexagonal array converging to a line-of- sight source. -----	14
Figure 16. Snapshots of the calculated acoustic pulse propagating from a point source with no scattering objects (on the left) and with a reflective corridor (on the right). -----	15
Figure 17. Time-reversed waves from the six-sensor array with and without the walls -----	16

INTRODUCTION AND APPROACH

Sound propagation in urban terrain is characterized by multiple propagation paths due to reflection and diffraction. These effects reduce the reliability of gunfire location systems based on beamforming or pulse arrival times, particularly for non-line-of-sight sources. If, however, the environment that causes the sound scattering (such as buildings) can be modeled on a computer and adequate sensor data can be collected, one can use time-reversal processing to model the propagation of a time-reversed waveform returning to the source. Time reversal has the added advantage that multiple propagation paths tend to enhance the fidelity of the returning wave.

The preliminary phase of the effort involved creating software to carry out finite-difference, time-domain (FDTD) modeling of two-dimensional sound propagation in a region containing a number of polygonal reflective structures, such as buildings and walls. The main thrust of the effort was then using this software to locate an impulsive sound source in various hypothetical urban environments using small sensor arrays and time-reversal processing. The focus was on identifying the capabilities and limitations associated with this approach, including the minimum workable array size. Particular attention was paid to locating a non-line-of-sight source (around the corner or on the other side of a building).

Time Reversal

In both acoustics and optics, the wave equation is symmetric with regard to time reversal:

$$\nabla^2 \psi - \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$
 (where ψ is an arbitrary function of space and time, t is time, and c is the speed of wave travel). Thus, if $\psi(x, y, z, t)$ is a solution of the wave equation, the time-reversed function $\psi(x, y, z, -t)$ is also a solution. The creation of time reversed acoustic waveforms and the use of these waveforms to refocus energy at the source is well documented.¹⁻⁵

In an idealized demonstration of time reversal, sound would first radiate from a source, as shown in Figure 1, propagate through a region containing objects or materials that scatter sound (including reflection and diffraction), and be recorded by a number of sensors surrounding the region. After the event, the signal from each sensor would be reversed in time and broadcast from emitters placed at the corresponding sensor locations. (Conveniently, for many applications, piezoelectric ultrasonic transducers can function both as sensors and emitters.)

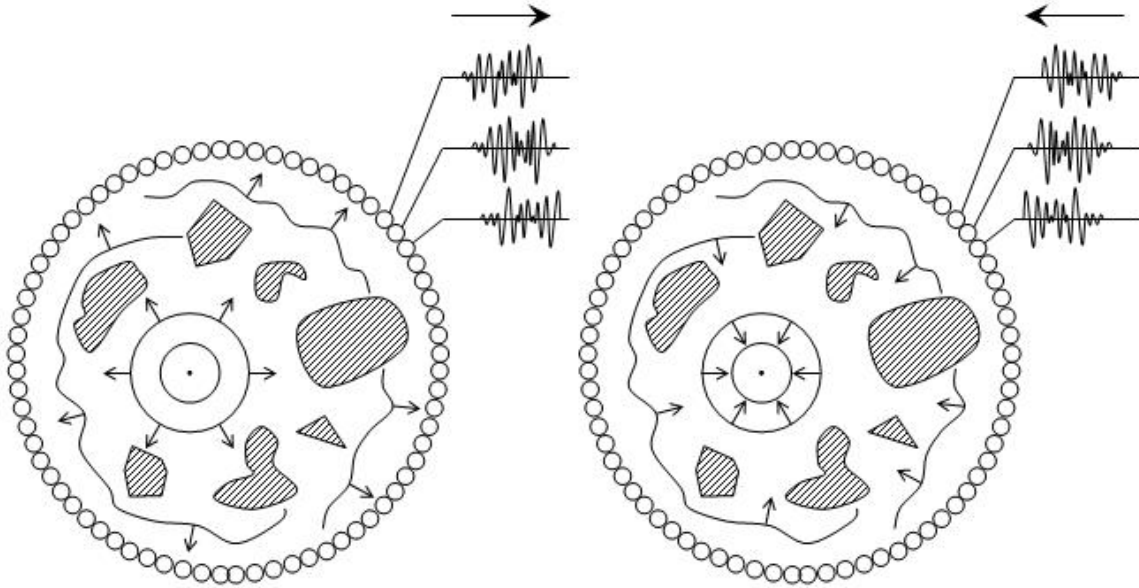


Figure 1. Idealized time reversal. On the left, sound radiates from a source, is scattered, and is recorded by closely spaced sensors surrounding the region. On the right, each sensor broadcasts a time-reversed signal, creating a time-reversed sound wave that returns to the original source.

According to the Huygens' principle, the waves from each emitter will superimpose to create a waveform identical to the original waveform in all respects except that the propagation vector will be at all points directed opposite to the propagation vector of the original wave. Further, because of the time symmetry of the wave equation, that reversed waveform will travel (and scatter) back through the region and return to the original source. The time-dependent sound pressure field throughout the region would be completely reversed as a function of time. Hence, the array of sensors combined with the processing to reverse the waveform is sometimes called a time-reversal mirror.

It should be emphasized that either portion of the above process, forward propagation or time-reversed propagation, can be made to occur either physically or numerically (computationally modeled on a computer). Thus *both* portions could occur physically, with actual sound traveling from a source to the sensors, being time reversed electronically, and then being broadcast from the sensors and sent back to the source. In this way, for example, high-energy ultrasound can be precisely focused to break up a gallstone or kidney stone.^{6,7}

In the process of locating a sound source in a physical environment, the forward propagation would occur physically, the sound would be picked up by sensors, and the time reversed propagation would then be modeled on a computer to discover where the reversed wave would focus. In this manner, time reversal has been demonstrated for locating gunfire in urban terrain⁸ as well as submarines in the ocean.⁹ These type of applications require a pre-existing computer model of the acoustic environment.

For an ideal time-reversal mirror, the sensors are closely spaced and completely surround the area containing the source. For a more practical time reversal mirror, the array of sensors subtends a smaller angle as viewed from the source, and we expect the fidelity of the time reversed wave to be reduced. If the reduced array were viewed as a traditional mirror, the diffraction-limited resolution would then be given approximately by $f\lambda/D$, where f is the focal length (distance to the source in this case), λ is the wavelength, and D is the aperture of the mirror (transverse size of the array).

Time reversal, however, makes use of *all* waves arriving at the sensors, including those arriving via multiple propagation paths due to scattering. Thus a wide angular distribution of scatterers (as viewed from the source) will direct waves toward the sensors that would otherwise, without the widely distributed scatters, not reach the sensors. As a result, multiple scattering typically leads to better than diffraction-limited resolution.^{10, 11} If the source is effectively immersed in a scattering environment, the array of sensors need not surround the environment (as stipulated by the ideal), but may be made fairly small. Similarly, one would expect it would be possible to reduce the number of sensors within an array of a given size and still maintain fidelity of time reversal to an acceptable level.

The Army's Cold Regions Research and Engineering Laboratory has studied time reversal to locate an impulsive sound source in urban terrain with three to eight sensors distributed throughout the environment.⁸ They used a 150 meter by 100 meter full-scale artificial urban training village with 15 concrete buildings (McKenna MOUT site, Ft. Benning, Georgia). They found that even with as few as three sensors, it may be possible to locate the source through time reversal processing. It must be noted, however, that although the sensors were few in number, they were widely spaced and effectively surrounded, or nearly surrounded, the source geometrically. The issue of how small the array (or aperture) might be made was not addressed.

The effort reported here, however, does address that issue. In this work, time reversal was attempted for relatively small sensor arrays, ten meters across or less, completely remote from the source rather than "enclosing" it in any way. As viewed from the source, then, the array aperture was relatively small.

Finite-Difference, Time-Domain Modeling

It is more straightforward to apply the finite-difference method to acoustics by starting with the continuity and Euler's equations rather than the wave equation.

In two dimensions, the linear continuity equation can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{1}{\rho c^2} \frac{\partial p}{\partial t} = 0,$$

where u is the particle velocity in the x -direction, v is the particle velocity in the y -direction, p is the acoustic pressure (instantaneous pressure minus the constant equilibrium pressure in the medium), ρ is the constant equilibrium density, and c is the speed of sound in the medium.¹²

The two components of the linear Euler's equation (also called the force equation) are given by¹²

$$\frac{\partial p}{\partial x} + \rho \frac{\partial u}{\partial t} = 0 \text{ and } \frac{\partial p}{\partial y} + \rho \frac{\partial v}{\partial t} = 0 .$$

These equations can then be written as the following difference equations:

$$\Delta p = -\rho c^2 \Delta t \left(\frac{\Delta u}{\Delta x} + \frac{\Delta v}{\Delta y} \right) \text{ (discrete continuity equation)}$$

$$\Delta u = -\frac{\Delta t}{\rho} \left(\frac{\Delta p}{\Delta x} \right) \text{ (x component of discrete Euler's equation)}$$

$$\Delta v = -\frac{\Delta t}{\rho} \left(\frac{\Delta p}{\Delta y} \right) \text{ (y component of discrete Euler's equation)}$$

For speed of computation, the finite-difference calculation is carried out using the so-called staggered Yee mesh.¹³ In this approach, pressures are calculated at discrete points on a square lattice and the particle velocities at points on shifted lattices, u shifted half a lattice vector in x and v shifted half a lattice vector in y , as shown in Figure 2.

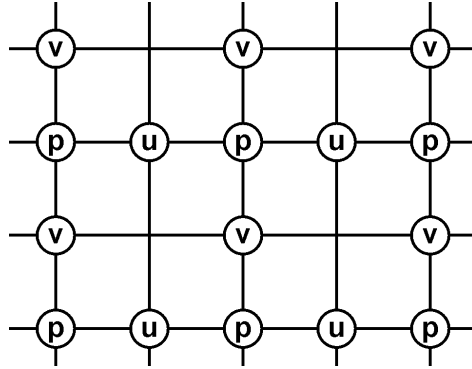


Figure 2. Two-dimensional staggered Yee mesh used in the finite-difference calculation. Pressures are calculated at lattice points labeled p. The particle velocities are calculated at points labeled u and v.

In addition, the pressure and particle velocity are calculated in alternating time steps. Thus, in one time step, the pressures are calculated from the adjacent velocity components (available from the previous step) via the discrete continuity equation. At the next time step, the velocity components u and v are calculated from the adjacent pressures via the discrete Euler's equation.

Reflections due to truncation of the domain at its outer edges are avoided by use of an absorbing, nonreflecting boundary based on Yuan et al.'s method¹⁴ of implementing Berenger's perfectly matched layer.¹⁵ The perfectly matched layer in this case has eight layers.

In finite difference processing, reflection, refraction, and diffraction occur as a result of spatial variations in the equilibrium density ρ and the speed of sound c . In this application, reflection and diffraction of sound at solid barriers such as walls and buildings is of primary concern, and the transmission of sound within such objects are ignored. Hence a vast simplification will be realized by assuming these solids do not support shear waves and are thus "fluid" in the acoustic sense. The change of acoustic impedance (ρc) at the air-solid interfaces will be sufficiently high that a negligible amount of transmitted acoustic energy re-emerges from the solids.

Because the high contrast of acoustic impedance at the air-solid interface can result in solution instability, one-lattice density averaging is used at those interfaces following Schröder and Scott.¹⁶

Reflection and diffraction at an interface (between two fluids, as discussed above) is determined by the change in acoustic impedance ρc at the interface. For simplicity, the acoustic impedance for the structures (walls and buildings) in these simulations will be assumed to be that of concrete, a highly reflective material with both a higher density and a higher sound velocity than air. For finite difference modeling, however, high sound velocities are undesirable due to the so-called Courant condition for stability. For a square lattice, the time step Δt must satisfy

$$\Delta t \leq \frac{\Delta x}{\sqrt{2}c_{\max}}$$

to guarantee numerical stability (where c_{\max} is the highest sound velocity in the model).¹⁷ (This condition can be intuitively understood as stating that during time-stepping, information must travel faster than sound.) A higher sound velocity in any region of the model thus requires a shorter time step (or larger lattice spacing). It is not the sound velocity per se that determines reflection and diffraction, however, but the product ρc . Hence the desired acoustic impedance for the structures can be achieved by keeping the sound velocity equal to that of air and compensating by assigning an artificially high density.

Thus 344 m/s, the sound velocity of dry air at 21° C, is used for both air and the structures. A density of 1.2 kg/m³ is used for air. The density of the structures is assumed to be 19,440 kg/m³, to give a structure-to-air impedance ratio of 16,200, similar to what would be given by the actual velocity and density of concrete (2950 m/s and 2300 kg/m³).

For the impulsive source, the model uses a point source of pressure given by the following waveform, which has been shown to reasonably approximate the pressure pulse from a gunshot or explosion:^{8, 18}

$$P(t) = A \left[1 - \left(\frac{ft - B}{B} \right)^2 \right] \exp(-Cft)$$

where A is an arbitrary amplitude, $B = 0.25$, $C=3.39$, and $f = 150$ Hz. To speed the calculation slightly, P is assumed to be zero after 0.025 s. This waveform is plotted in Figure 3.

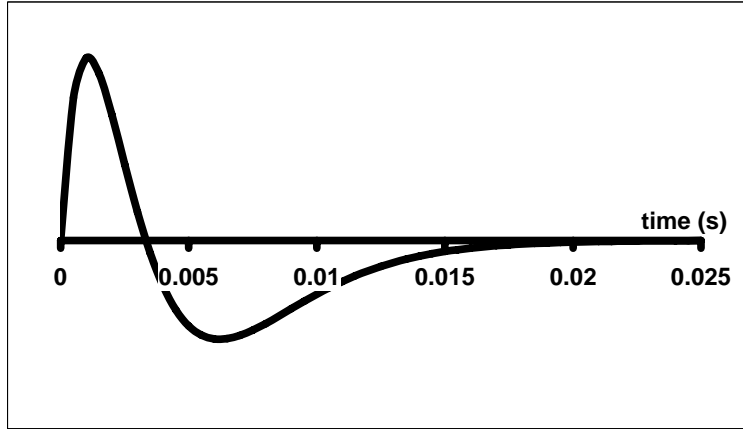


Figure 3. Waveform used for impulsive sound source.

For the finite-difference calculations, the spacing between adjacent pressure points (and adjacent velocity points) was 0.1 m, less than 1/20 of the nominal wavelength of the source function. The calculations used a time step of 0.1 ms between successive pressure calculations (and between successive velocity calculations).

The calculations were carried out using MATLAB. For evaluation and visualization purposes, each run of the finite-difference calculations produced a video file (AVI) of the pressure field evolution and a file containing pressure versus time for various sensor positions.

RESULTS

Figure 4 shows the calculated acoustic pulse propagating from a point source in one hypothetical urban area that was studied. Although other hypothetical regions were studied, this region is convenient for illustrating a number of typical results. In all cases shown for this region, each calculation was carried out for 350 ms of sound travel to ensure the complete waveform and reflections were received at the sensors.

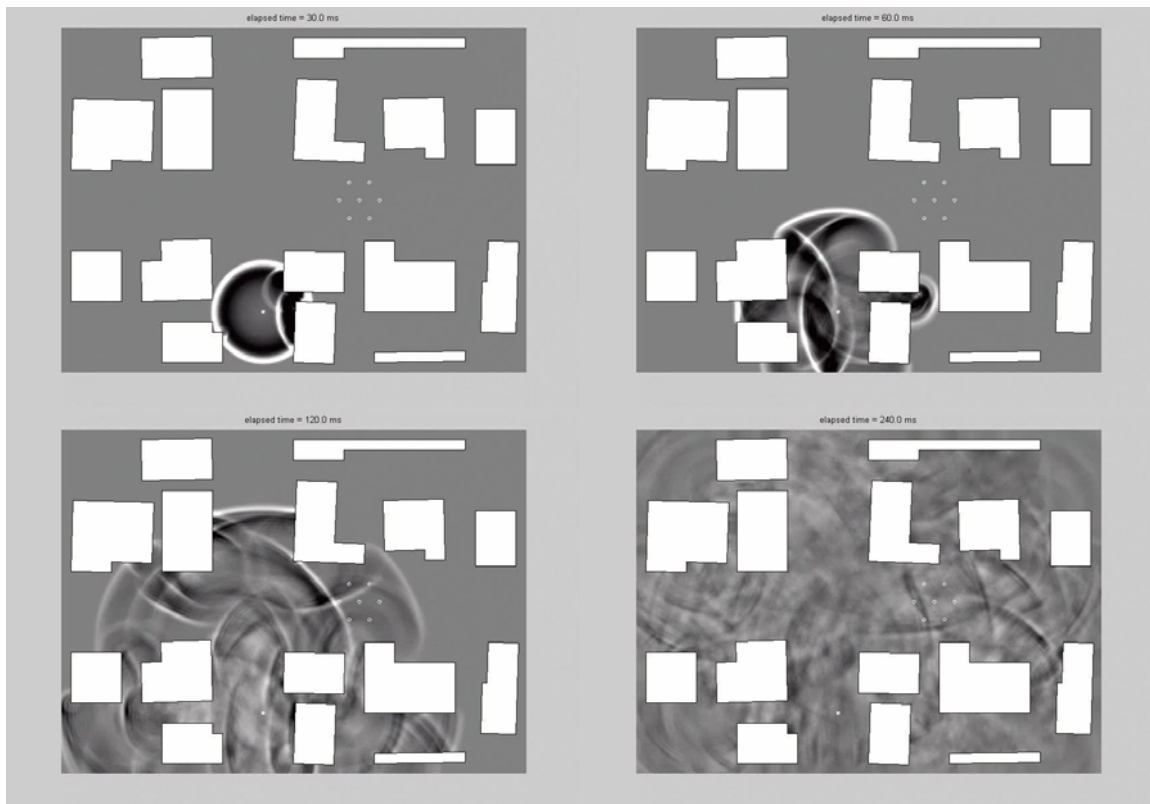


Figure 4. Calculated acoustic pulse propagating from a point source in a 92 meter by 68 meter hypothetical urban area. The four images are snapshots of the sound pressure at different times. Buildings are white and open areas are gray. Positive sound pressure is depicted as white and negative pressure as black. The source is shown as a single white circle at the bottom of the vertical “street.” A seven-sensor hexagonal array eight meters across is shown as seven white circles in the horizontal “street.”

The images in Figure 4 also show a seven-sensor hexagonal array eight meters across (two meters between adjacent sensors). The pressure traces were recorded at these sensor positions for the subsequent time reversal processing shown in Figure 5. The two lower images of Figure 5 show the time-reversed wave converging at the location of the original source, focusing at the bright white spot in the lower right image. Note that time reversal with this hexagonal array was able to locate a non-line-of-sight source. This array, the largest studied, generally produced good results.

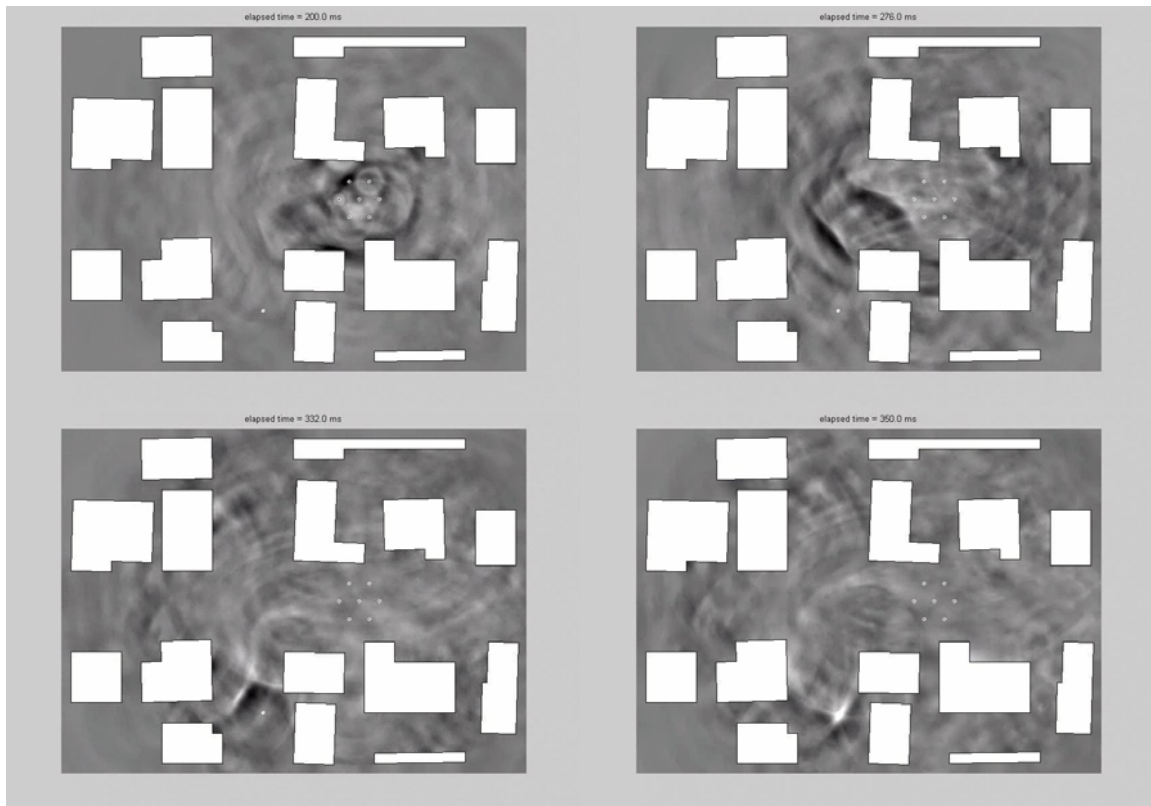


Figure 5. Time-reversed sound from seven-sensor hexagonal array. The two lower snapshots show the time-reversed wave converging at the location of the original source, focusing at the bright white spot in the lower right image. (Source as in Figure 4.)

Figure 6 shows four snapshots of the time-reversed pressure field for the same point source as in Figure 4 but with an array of two small subarrays ten meters apart. Each subarray consisted of four sensors with one at the center and the other three each one meter from the central sensor and arranged on an equilateral triangle. Again, the lower right image shows the time-reversed sound focusing at the original source location.

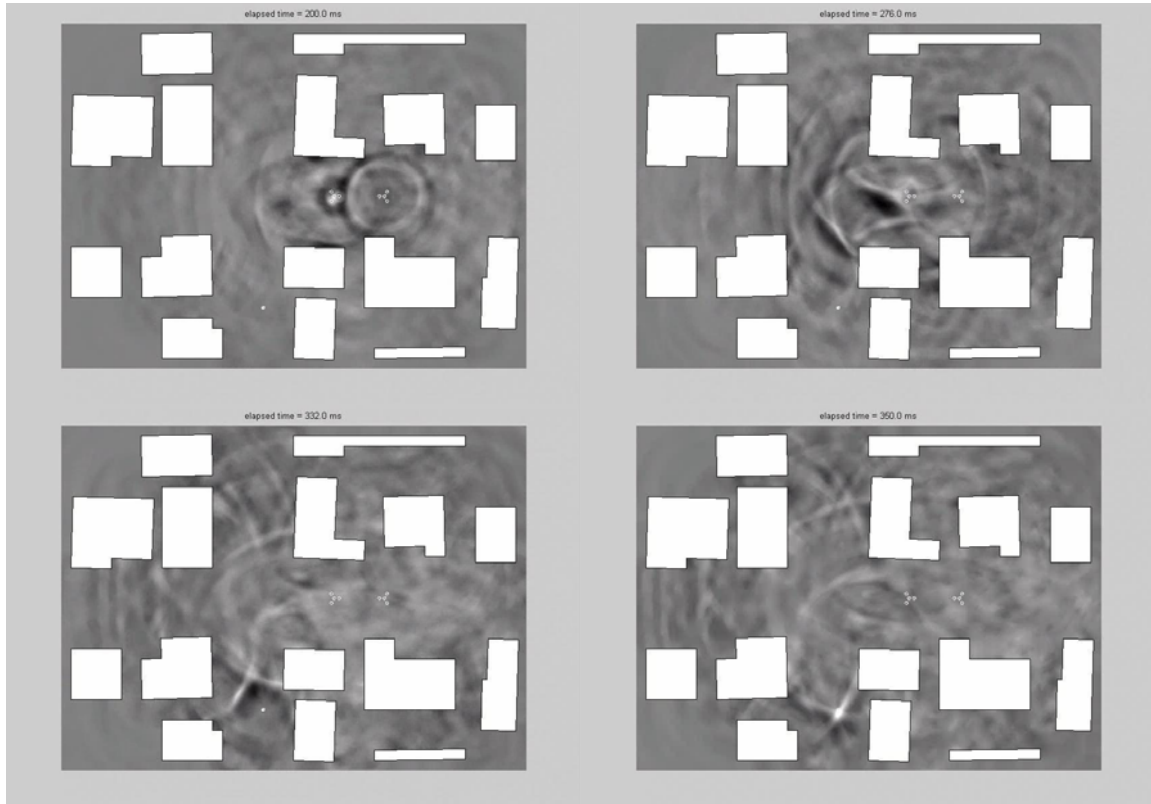


Figure 6. Time-reversed sound from an array of two small subarrays ten meters apart. The two lower snapshots show the time-reversed wave converging at the location of the original source, focusing at the bright white spot in the lower right image. (Source as in Figure 4.)

Figure 7 shows four snapshots of the time-reversed pressure field for the same point source as in Figure 4 but with a five-sensor portion of the hexagonal array close to the wall of a building which acts as a mirror for sound.

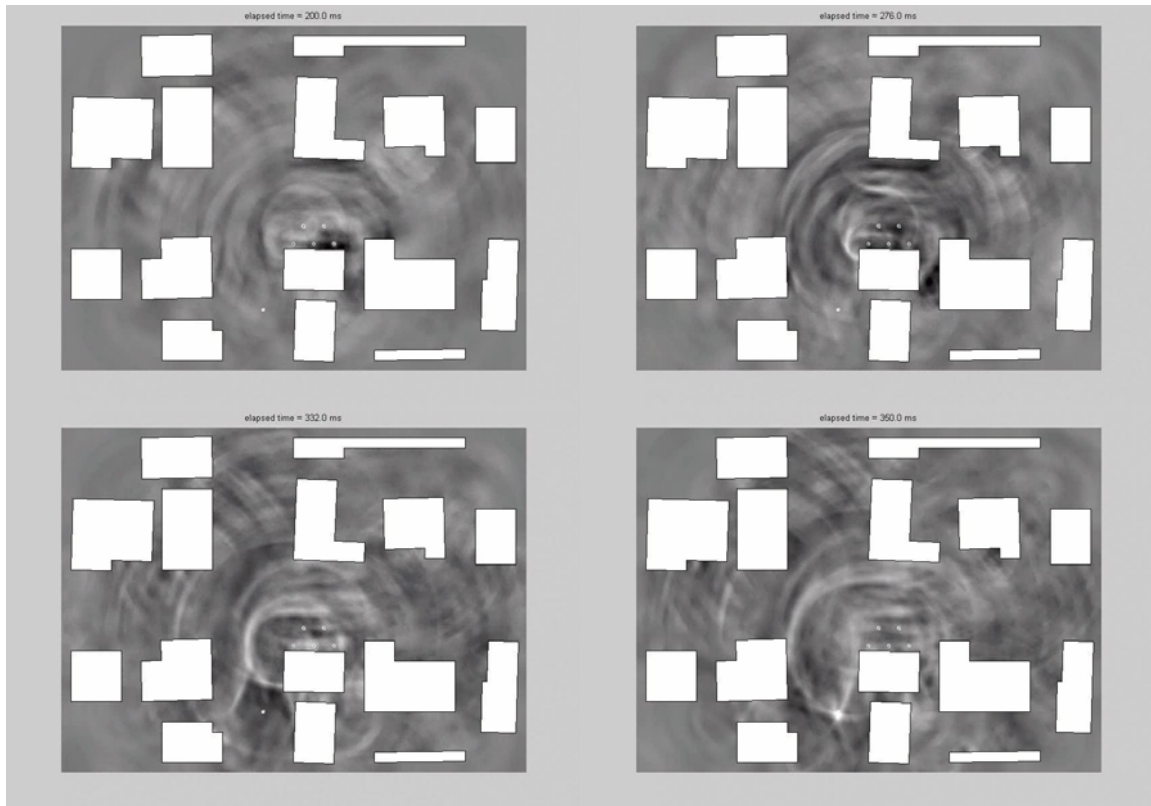


Figure 7. Time-reversed sound from a five-sensor portion of the seven-sensor hexagonal array backed against a wall. The two lower snapshots show the time-reversed wave converging at the location of the original source, focusing at the bright white spot in the lower right image. (Source as in Figure 4.)

Figure 8 shows the calculated acoustic pulse propagating from a point source at a *different* location from previous examples.

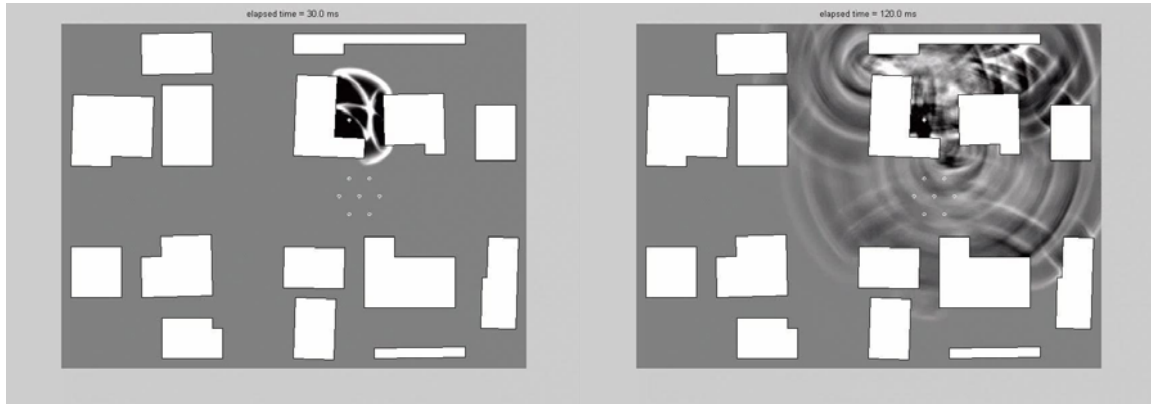


Figure 8. Calculated acoustic pulse propagating from a point source at a different location from previous examples.

Figures 9 and 10 show the time-reversed pressure field for two different sensor arrays using the sound from the source at the second location.

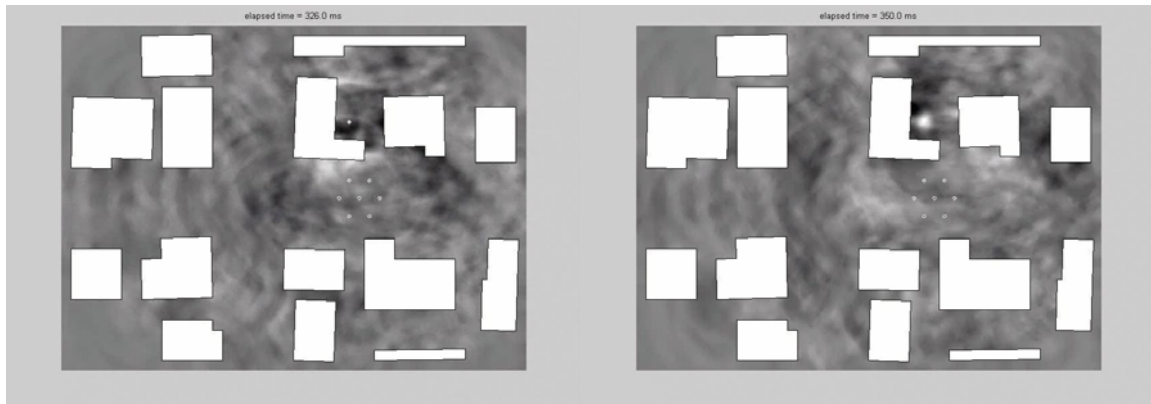


Figure 9. Time-reversed sound from the seven-sensor hexagonal array. The two snapshots show the time-reversed wave converging at the location of the source. (Source as in Figure 8.)

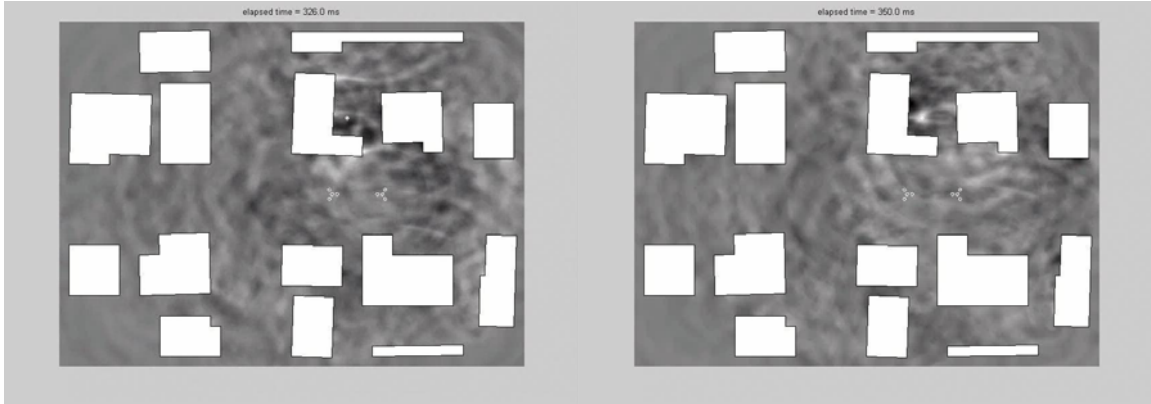


Figure 10. Time reversed sound from the array of two small subarrays ten meters apart. (Source as in Figure 8.)

Figure 11 shows the calculated acoustic pulse propagating from a point source at a *third* location.



Figure 11. Calculated acoustic pulse propagating from a point source at a third location.

Figures 12 and 13 show the time-reversed pressure field for two different sensor arrays using the sound from the source at the third location.

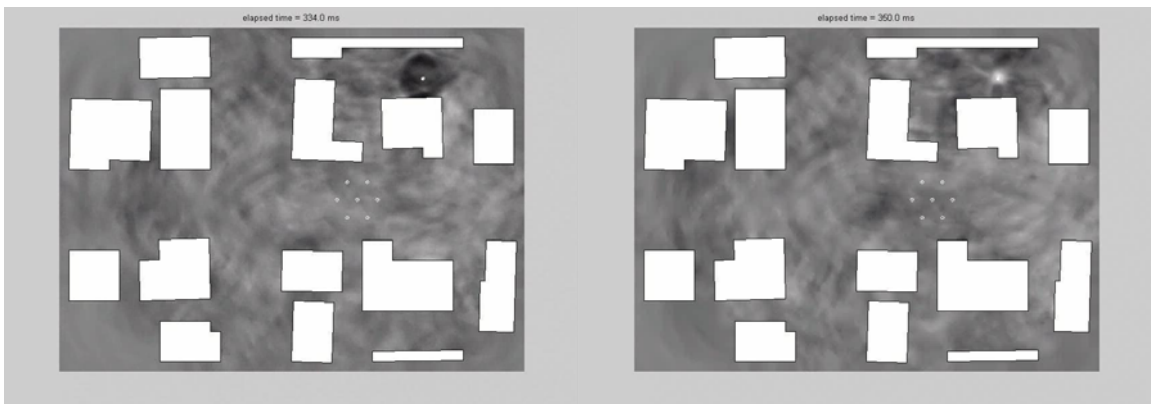


Figure 12. Time-reversed sound from the seven-sensor hexagonal array. (Source as in Figure 11.)

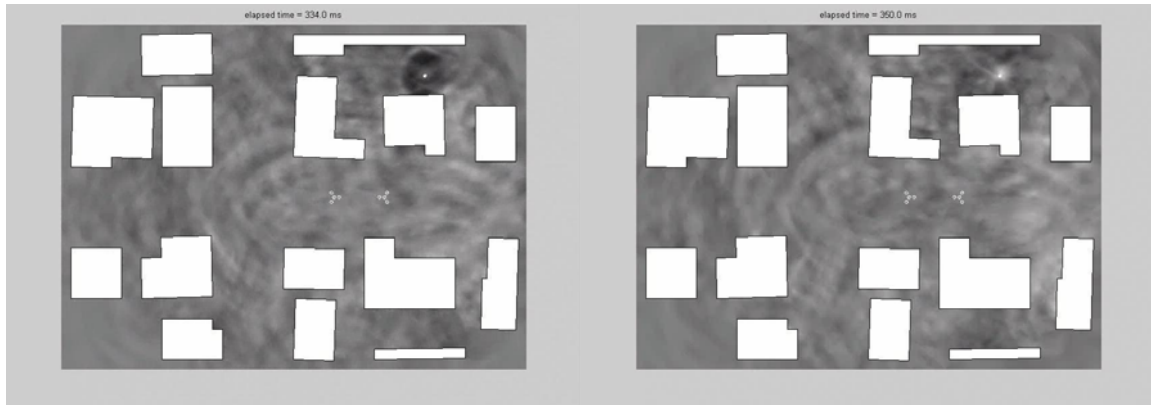


Figure 13. Time-reversed sound from the array of two small subarrays ten meters apart. (Source as in Figure 11.)

The previous examples have all been of sensor arrays locating non-line-of-sight sources. In locating non-line-of-sight sources, however, there must be a reflective geometry that provides reasonably strong sound propagation from the source to the sensors. Hence an array on one side of a building would have difficulty locating a source on the opposite side unless there were additional buildings beyond the first to reflect sound toward the sensor area.

The next two figures illustrate the location of a direct, line-of-sight source using the seven-sensor hexagonal array. Figure 14 shows the calculated acoustic pulse propagating from the point source located at the left of the region.

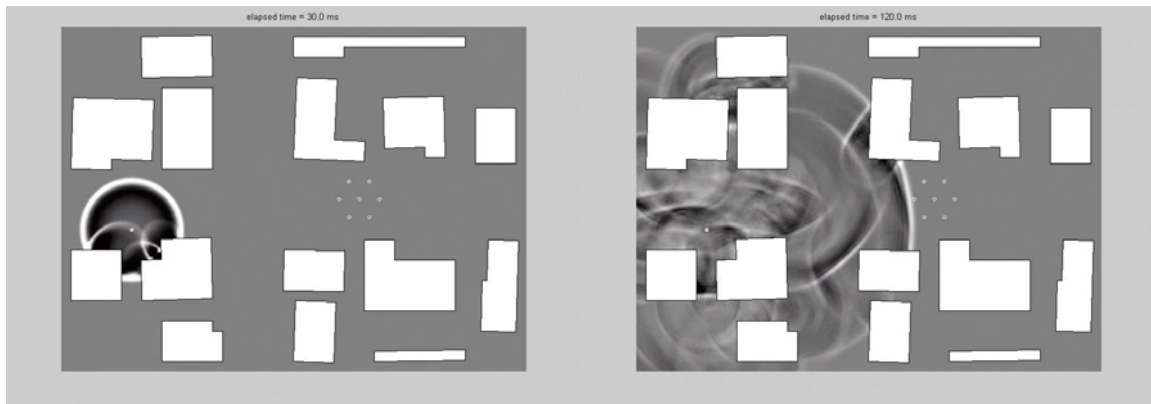


Figure 14. Calculated acoustic pulse propagating from a line-of-sight point source.

Figure 15 then shows snapshots of the time-reversed wave approaching and converging at the original source.

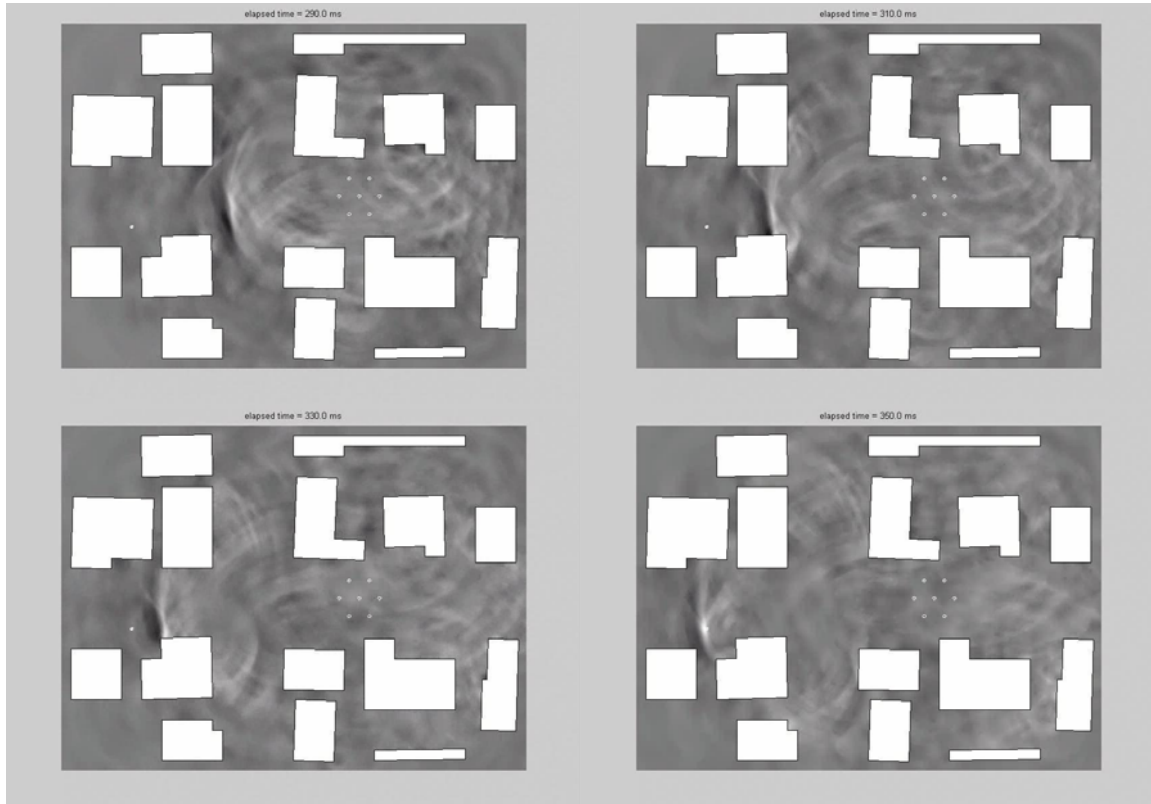


Figure 15. Time-reversed sound from the seven-sensor hexagonal array converging to a line-of-sight source. (Source as in Figure 14.)

Figures 16 and 17 illustrate how multiple propagation paths enhance time reversal. The left side of Figure 16 shows a sequence of snapshots of the calculated acoustic pulse propagating from a point source with no objects to scatter sound. The right side shows the calculated acoustic pulse propagating from an identical point source but with a reflecting corridor formed by two parallel walls. In both cases, a linear array of six sensors is located at the right of the region, 40 meters from the source and 2 meters between adjacent sensors. The walls are 29 meters long and 16 meters apart.

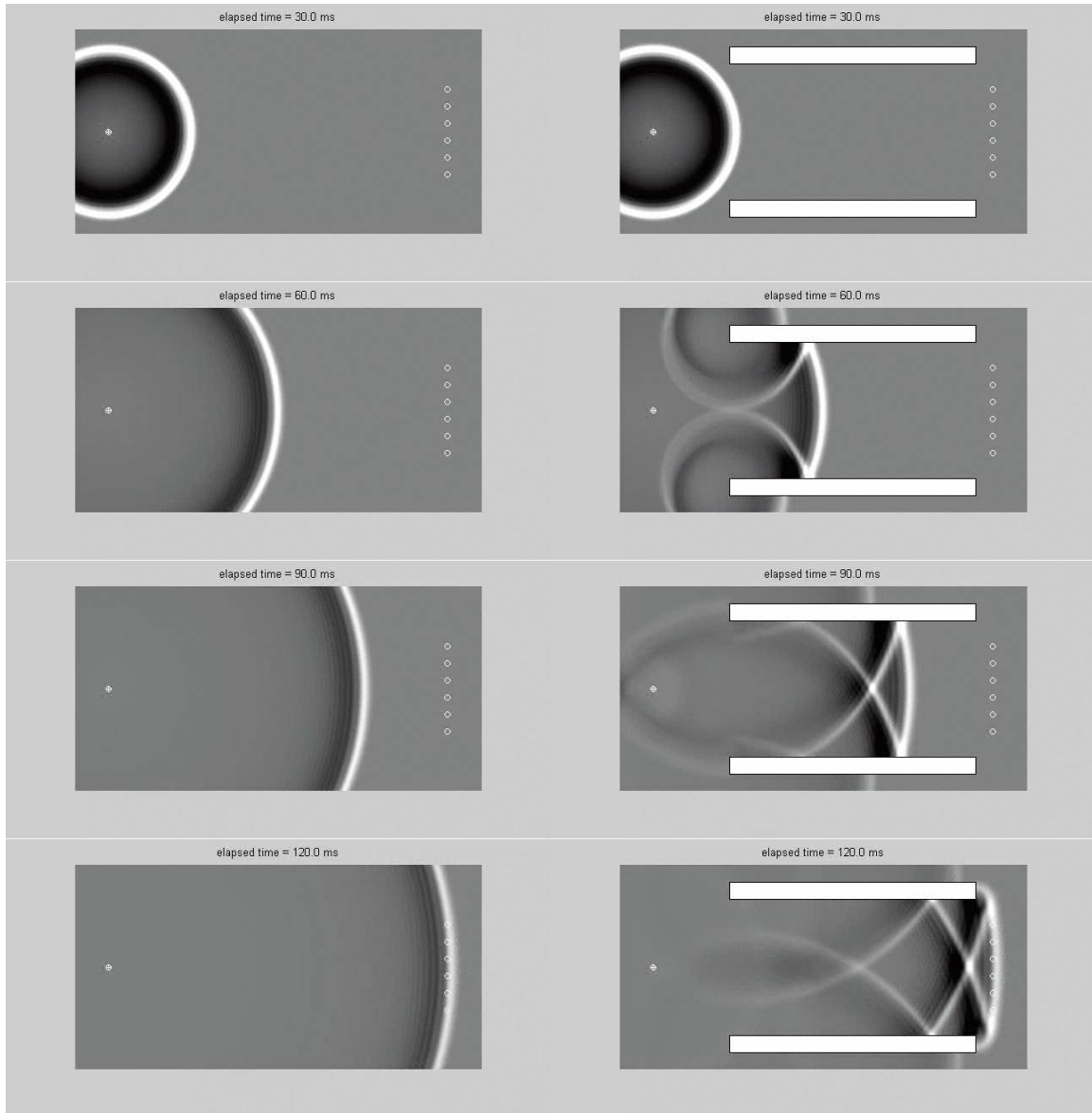


Figure 16. Snapshots of the calculated acoustic pulse propagating from a point source with no scattering objects (on the left) and with a reflective corridor (on the right). A linear array of six sensors is at the right of both regions.

Figure 17 shows the time-reversed waves from the six-sensor array with and without the walls. In the presence of the walls, the sound refocuses much more tightly at the original source location.

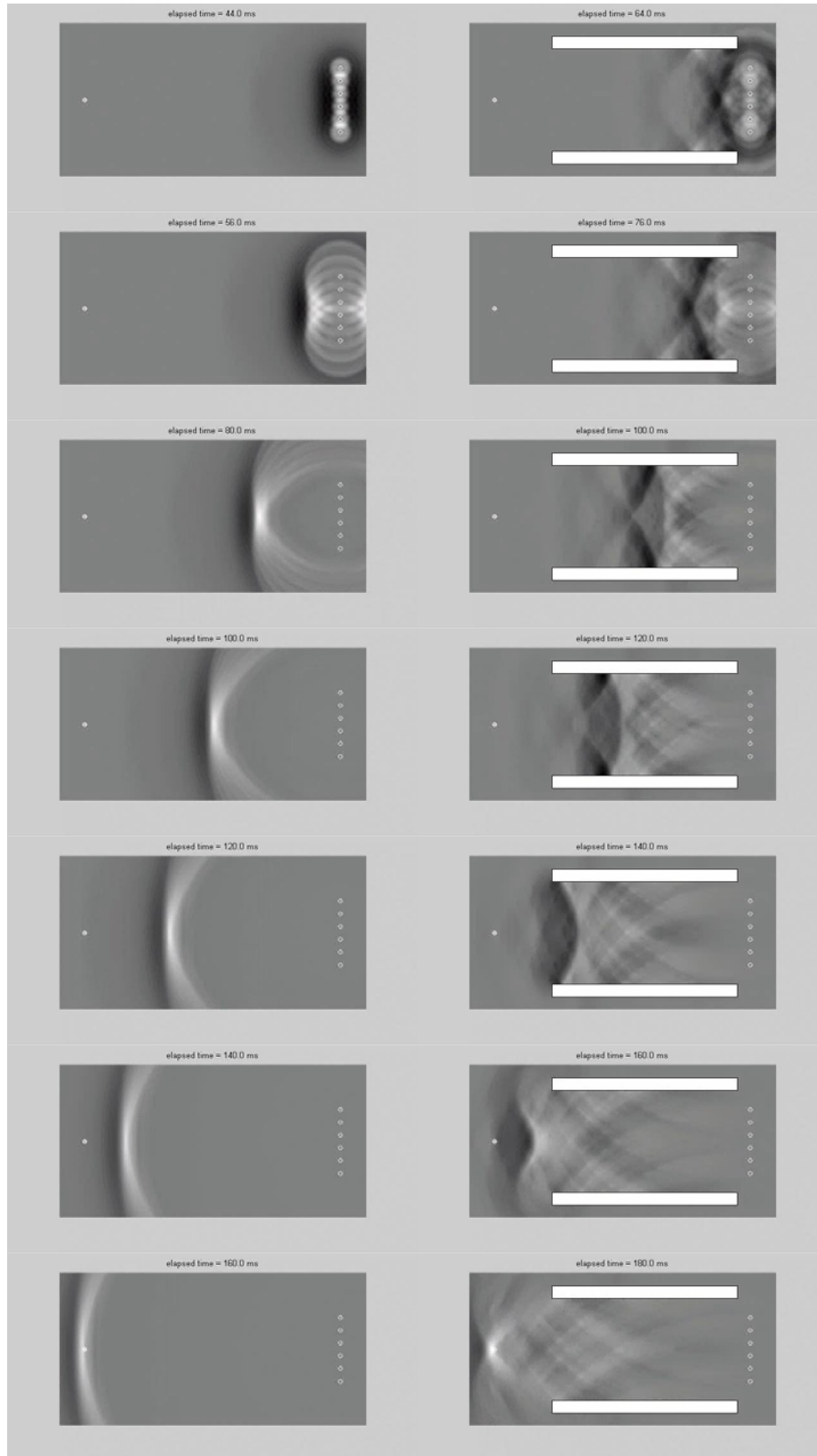


Figure 17. Time-reversed waves from the six-sensor array with and without the walls. The sound refocuses much more tightly with the walls present.

As stated earlier, the array of sensors combined with time-reversal processing is sometimes called a time-reversal mirror. If the reduced array is viewed as a traditional mirror, the diffraction-limited resolution is approximately $f\lambda/D$, where f is the distance to the source, λ is the wavelength, and D is the transverse size of the array. The corridor walls, however, direct sound toward the sensors that would otherwise be lost. Hence, the walls effectively increase the aperture D of the time-reversal mirror, thus reducing the size of the refocused spot.

Consistent with that interpretation, it was generally observed that adding buildings and reflective propagation paths tended to enhance the ability of time reversal to locate the source.

CONCLUSION

The use of time reversal processing with small sensor arrays to locate impulsive sound sources was studied by calculating the forward and time-reversed propagation of sound in various hypothetical urban environments. The focus was on relatively small sensor arrays, ten meters across or smaller, completely remote from the source rather than “enclosing” it in any way, a scenario far from ideal for time reversal. Although this leads to poor fidelity of the overall time-reversed wave, it is still often possible to determine the location of the source from a video animation.

Generally speaking, the results confirmed that the ability to locate a source decreases as the number of sensors and the spatial extent of the array is reduced. Part of the purpose of the investigation was to learn just how small an array could be made yet still be able to locate a source, particularly non-line-of-sight. Arrays of seven or more sensors with a total spatial extent of eight to ten meters were sufficient for locating non-line-of-sight sources, whereas smaller arrays proved inadequate. A seven-sensor hexagonal array eight meters across, for example, performs quite well in locating a source in urban terrain, even on the opposite side of a building. A more practical array consisting of two smaller four-sensor subarrays ten meters apart is still adequate for non-line-of-sight and around-the-corner source location. (Because the distance between subarrays is not critical, so long as it is approximately ten meters or more, each subarray can conceivably be carried on a vehicle.)

As the size of the arrays is reduced below eight to ten meters, the performance degrades significantly. A single small array of three or four sensors spaced one to two meters apart, for example, is generally insufficient to locate a source.

One practical drawback of the FDTD method is its long processing time, even in two dimensions. For a typical 90×70 square meter area modeled in this study, a 3.6 GHz personal computer took 15 minutes to process 350 milliseconds of sound propagation.

Given the high degree of reflection in an urban environment, however, it may be possible to neglect diffraction when using time reversal to locate the source. Hence a less computationally intense approach based on geometrical acoustics, such as beam tracing,^{19, 20} may be feasible. A beam tracing approach could make time reversal practical for source location (1) with small

sensor arrays as discussed in this report; (2) with synchronized sensors distributed throughout a town (already demonstrated by others using time reversal with FDTD processing); and (3) with three-dimensional modeling for rooftop or tower-based sensor arrays.

Although the results presented here are not from actual physical measurements, they do illustrate the *potential* that time reversal holds for source location.

REFERENCES

1. M. Fink, "Time reversal of ultrasonic fields – Part I: Basic principles" *IEEE Trans. on Ultrasonics, Ferroelectrics, and Frequency Control* **39**, 555-566 (1992).
2. F. Wu, J.-L. Thomas, and M. Fink, "Time reversal of ultrasonic fields – Part II: Experimental results," *IEEE Trans. on Ultrasonics, Ferroelectrics, and Frequency Control* **39**, 567-578 (1992).
3. A. Derode, P. Roux, and M. Fink, "Robust acoustic time reversal with high order multiple scattering," *Phys. Rev. Lett.* **75**, 4206-4209 (1995).
4. M. Fink, "Time-reversed acoustics," *Physics Today* **50**, no. 3, 34-40 (1997).
5. M. Fink, G. Montaldo, and M. Tanter, "Time reversal acoustics," *2004 IEEE Ultrasonics Symposium* **2**, 850-859 (2004).
6. J.-L. Thomas, F. Wu, and M. Fink, "Time reversal focusing applied to lithotripsy," *Ultrasonic Imaging* **18**, 106–121 (1996).
7. M. Fink, D. Cassereau, A. Derode, C. Prada, P. Roux, M. Tanter, J.-L. Thomas, F. Wu, "Time-reversed acoustics," *Reports on Progress in Physics* **63**, 1933-1995 (2000)].
8. D. G. Albert, L. Liu, M. L. Moran, "Time reversal processing for source location in an urban environment," *J. Acoust. Soc. Am.* **118**, 616-619 (2005).
9. W. A. Kuperman, W. S. Hodgkiss, H. C. Song, T. Akal, C. Ferla, D. R. Jackson, "Phase conjugation in the ocean: Experimental demonstration of an acoustic time-reversal mirror," *J. Acoust. Soc. Am.* **103**, 25-40 (1998).
10. A. Derode, P. Roux, M. Fink, "Robust acoustic time reversal with high-order multiple scattering," *Physical Review Letters* **75**, 4206-4209 (1995).
11. A. Derode, A. Tourin, M. Fink, "Time reversal in multiply scattering media," *Ultrasonics* **36**, 443-447 (1998).
12. L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of Acoustics*, 4th ed., John Wiley & Sons (1999).
13. K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Transactions on Antennas and Propagation* **AP-14**, 302-307 (1966).
14. X. Yuan, D. Borup, J. W. Wiskin, M. Berggren, R. Eidens, and S. A. Johnson, "Formulation and validation of Berenger's PML absorbing boundary for the FDTD

- simulation of acoustic scattering,” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control* **44**, 816-822 (1997).
15. J-P. Berenger, “A perfectly matched layer for the absorption of electromagnetic waves,” *Journal of Computational Physics* **114**, 185-200 (1994).
 16. C. T. Schröder and W. R. Scott, Jr., “On the stability of the FDTD alorithm for elastic media at a material interface,” *IEEE Transactions on Geoscience and Remote Sensing* **40**, 474-481 (2002).
 17. A. Taflove, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, Boston, Artech House (1995).
 18. L. Liu, D. G. Albert, “Acoustic pulse propagation near a right-angle wall,” *J. Acoust. Soc. Am.* **119**, 2073-2083 (2006).
 19. T. Funkhouser, I. Carlbom, G. Elko, G. Pingali, M. Sondhi, and J. West, “A beam tracing approach to acoustic modeling for interactive virtual environments,” *SIGGRAPH '98 - Proceedings of the 25th Annual Conference on Computer Graphics and Interactive Techniques*, 21-32 (1998).
 20. T. Funkhouser, N. Tsingos, I. Carlbom, G. Elko, M. Sondhi, J. West, G. Pingali, P. Min, and A. Ngan, “A beam tracing method for interactive architectural acoustics,” *J. Acoust. Soc. Am.* **115**, 739-756 (2004)